## [**Chapter 1 Solutions**](http://docs.google.com/chap1.html)

[**Chapter 2 Solutions**](http://docs.google.com/chap2.html)

[**Chapter 3 Solutions**](http://docs.google.com/chap3.html)

[**Chapter 5 Solutions**](http://docs.google.com/chap5.html)

## Exercise 1.2

### Recurrent Relation:

\(A(0)=0 \\ A(n)=3A(n-1)+2, \text{for }n\ge0\)

### Closed Form Hypothesis:

\(A(n)=3^n-1\)

### Inductive Proof:

\(A(0)=3^0-1=0 \\ A(n)=3(3^{n-1}-1)+2=3^n-1\)

## Exercise 1.2 Alternative - Generating Functions

\(\text{Recurrent relation} \\ A\_0=0 \\ A\_{n+1}=3A\_n+2, \text{for }n\ge0 \\ \text{Define the generating function} \\ G(x)=\sum\_{n\ge0}A\_nx^n \\ \text{Express }A\_{n+1}\text{ in terms of }G(x) \\ \sum\_{n\ge0}A\_{n+1}x^n=A\_1+A\_2x+a\_3x^2+...=\{A\_1x+A\_2x^2+a\_3x^3+...\}/x \\ \sum\_{n\ge0}A\_{n+1}x^n=G(x)/x \\ \text{Express }3A\_{n}+2\text{ in terms of }G(x) \\ \sum\_{n\ge0}(3A\_{n}+2)x^n=3(\sum\_{n\ge0}A\_{n}x^n)+(\sum\_{n\ge0}2x^n)\\ \sum\_{n\ge0}(3A\_{n}+2)x^n=3G(x)+\frac2{1-x},\text{ for }x<1 \\ \text{Solve }G(x) \\ G(x)/x=3G(x)+\frac2{1-x},\text{ for }x<1 \\ G(x)-3xG(x)=\frac{2x}{1-x}=(1-3x)G(x) \\ G(x)=\frac{2x}{(1-x)(1-3x)} \\ \text{Partial Fractions Expansion} \\ \frac{2x}{(1-x)(1-3x)}=\frac{y}{1-x}+\frac{z}{1-3x} \\ 2x=(1-3x)y+(1-x)z\\ (1-3(\frac13))y+(1-\frac13)z=\frac23z=\frac23 \\ 2x=(1-3x)y+(1-x)\\ y=\frac{-1+3x}{1-3x}=-1 \\ G(x)=\frac{1}{1-3x}-\frac{1}{1-x},\text{ for }x<1 \\ \text{Expand }G(x)\text{ into a power series} \\ G(x)=\sum\_{n\ge0}(3x)^n - \sum\_{n\ge0}x^n= \sum\_{n\ge0}3^nx^n - \sum\_{n\ge0}x^n \\ [x^n]G(x)=A\_n=3^n-1 \)

## Exercise 1.6

### Recurrent Relation:

\(B(2)=0 \\ B(n)=B(n-1)+n-2,\text{ for }n>2\)

### Unroll The Recurrence:

\(B(n)=B(n-2)+(n-1-2)+(n-2) \\ B(n)=0+(3-2)+...+(n-1-2)+(n-2) \\ B(n)=0+(n-2)+...+(4-2)+(3-2) \\ 2B(n)=(n+3-4)(n-2)\)

### Closed Form Hypothesis:

\(B(n)=\frac{(n-1)(n-2)}2,\text{ for }n\ge2\)

### Inductive Proof:

\(\frac{(2-1)(2-2)}2=0 \\ \frac{(n-1)(n-2)}2=\frac{(n-2)(n-3)}2+n-2 \\ \frac{n^2-3n+2}2=\frac{n^2-3n+2}2\)

## Exercise 1.8

\(n=5m+l,\text{ where }0\le l\le4\text{ and } m \in \Bbb N \\ Q(5m+l)=P(l) \\ P(0)=\alpha \\ P(1)=\beta \\ P(2)=\frac{1+\beta}\alpha \\ P(3)=\frac{1+\beta+\alpha}{\alpha\beta} \\ P(4)=\frac{1+\alpha}\beta\)

## Exercise 1.11a

### Recurrent Relation:

\(D(0)=0 \\ D(2n)=2D(2n-2)+2, \text{for }n\ge0\)

### Closed Form Hypothesis:

\(D(2n)=2^{n+1}-2\)

### Inductive Proof:

\(D(2(0))=2^{0+1}-2=0 \\ s = 2n-2 \\ D(s) = 2^{(s/2)+1}-2 \\ D(2n)=2D(s)+2=2^{(s/2)+2}-4+2=2^{n+1}-2\)

## Exercise 13

### Recurrent Relation:

\(Z(0)=1 \\ Z(n)=Z(n-1)+9n-8,\text{ for }n>0\)

### Unroll The Recurrence:

\(Z(n)=Z(n-2)+(9(n-1)-8)+(9n-8) \\ Z(n)=1+(9(1)-8)+(9(2)-8)+...+(9n-8) \\ Z(n)=1+(9n-8)+(9(n-1)-8)+...+(9-8) \\ 2Z(n)=2+n(9(n+1)-16)\)

### Closed Form Hypothesis:

\(Z(n)=\frac{n(9n-7)}2+1,\text{ for }n>0\)

### Inductive Proof:

\(Z(1)=\frac{1(9(1)-7)}2+1=2 \\ Z(n)=\frac{(n-1)(9(n-1)-7)}2+1+9n-8 \\ Z(n)=\frac{(n-1)(9n-16)}2+9n-7 \\ Z(n)=\frac{9n^2-7n+2}2 \\ Z(n)=\frac{n(9n-7)}2+1\)

## Exercise 1.15

### Recurrent Relation:

\(I(1)=0 \\ I(2)=2 \\ I(2n)=2I(n)-1,\text{ for }n>0 \\ I(2n+1)=2I(n)+1,\text{ for }n>0\)

### Closed Form Hypothesis:

\(n=3(2^m)+l,\text{ where }0\le l<3(2^m)\le n \\ I(n)=2l+1,\text{ for }n>2\)

### Inductive Proof:

Basis:\(I(3)=I(3(2^0)+0)=2(0)+1=1\)

Even:\(I(n)=2(2(l/2)+1)-1=2l+1,\text{ for }n>2\)

Odd:\(I(n)=2(2(l-1)/2+1)+1=2l+1,\text{ for }n>2\)